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13. ABSTRACT (Maximum 200 words)

The primary activities supported by the grant during the reporting period include a new result showing the hardness of the single-source robust network design and an invitation to include this in the special issue devoted to selected papers in FOCS 2005. A summer intern was hosted, Andrew McGregor from UPenn, who worked with Shepherd on recognizing Hilbert Bases and other theoretical topics in Math Programming. A visit was also supported for Gianpaolo Oriolo (Rome), which resulted in some new joint work on robust network design. In addition, there was a week visit from Seffi Naor (Technicion). Travel supported during this period includes trips by Shepherd to UPenn to work with Sanjeev Khanna and C. Chekuri on the mutlicommodity flow problem. This work has resulted in the FOCS 2005 paper, which in addition was invited into a special issue of selected papers. Conferences attended were the 2004 APPROX/RANDOM (Chekuri) and CORC 4th Optimization Day (Shepherd).

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Fundamentals of Combinatorial Optimization and Algorithm Design: December Report

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The main activities being supported under this grant, are research support (primarily for travel) for C. Chekuri, B. Shepherd, and P. Winkler. Second to this, funds were used to support one summer intern, and several brief visits from scientists. Peter Winkler has now gone (as of July) on leave to a position at Dartmouth University, so he will no longer request further support from this grant. Part of his travel support will subsidize Shepherd's attendance at a meeting in Aussois and then to attend the International Network Optimization Conference (INOC).

Research **highlights** in the June-December period include a new result showing the hardness of single-source robust network design, and an invitation to include [3] in the special issue devoted to selected papers from FOCS 2005.

To date, we have hosted 1-2 week (Summer) visits from Anupam Gupta (CMU) and Adrian Vetta (McGill). We also had a week visit from Seffi Naor (Technion) and a subsequent 2-day follow up meeting to work with C. Chekuri in February. We hosted a summer intern Andrew McGregor from UPenn (under the supervision of Sampath Kannan) who worked with Shepherd on recognizing Hilbert Bases and other theoretical topics in Math Programming. Finally, we supported a visit by Gianpaolo Oriolo (Rome) which resulted in some new joint work on robust network design.

Travel support to date has consisted of funding Chekuri and Shepherd to attend (last May) a interesting workshop in Bertinoro, Italy. This was organized by Leen Stougie, Michel Goemans and Martin Skutella and brought together researchers in CS, OR and combinatorics. Chekuri spoke on work of himself and Shepherd on integer decomposition results for Steiner Forests and related network design problems [4].

We briefly summarize the work in progress. Please contact Bruce Shepherd (bshep@lucent.com) as we are very happy to provide some of the work in progress manuscripts.

Work on Edge Disjoint Paths of Chandra Chekuri, Sanjeev Khanna, Bruce Shepherd

The authors continue their study of the maximum edge-disjoint path problem (EDP). The recent highlights are that their FOCS 2005 paper was invited to the special issue of best papers from that conference. The authors have also recently (end of January) learned that their subsequent submission to STOC 2005 has been accepted.

In EDP we are given a graph G and terminals s_i, t_i and we wish to find a maximum collection of s_i, t_i pairs that admit a collection of edge disjoint paths. EDP is one of the most fundamental problems in combinatorial optimization. Apart from its applications in VLSI design and network design, it is also intrinsic to many approaches for solving other applied problems such as scheduling. For instance, Shepherd and Matthew Andrews created a scheduling system based

on EDP that went into a Lucent product in the late 1990's.

In [1] it was shown that in directed graphs it is hard to approximate the optimum to within a factor of $m^{.5-\epsilon}$ for any $\epsilon > 0$, and hence EDP is terribly difficult with respect to polytime approximations. Their reduction however breaks down if small edge congestions are allowed, and also breaks down for undirected graphs. In fact, the gap for undirected graphs is only known to lie in the range 2 to $m^{.5-\epsilon}$. In [2], we show the first positive results in this direction by showing that a linear relaxation (the all-or-nothing multicommodity flow problem) problem does admit a poly-logarithmic (i.e., polynomial in the variable $\log(n)$) approximation for general graphs – please refer to the paper: `http://cm.bell-labs.com/cm/ms/who/bshep/pub.html`. Those techniques used recent results of Räcke on oblivious routing as well as some interesting graph theoretic clustering results in 2-connected graphs. The paper inspired the authors to take a look at EDP itself, based on an insight from [3]. Using schemes of Robertson, Seymour and Thomas, they have been able to prove the following theorem in planar graphs. We call a set X *well-linked* in G, if for any subset S with $|S \cap X| \leq |X - S|$, we have $|\delta(S)| \geq |S \cap X|$. We show that if S is well-linked in a planar graph G, there is a constant C (about $10,000$ at present unfortunately) such that for any matching M of size $|S|/C$ on S, we can find paths connecting the endpoints of M, such that each edge lies in at most 2 of them. This result implies (again using Räcke's results) that EDP can be approximated to within a polylogarithmic factor where only $\sqrt{n}$ was previously best known. The result also shows that there is a constant approximation for product multicommodity flow in planar graphs, thus giving a new proof of an earlier result of Klein, Plotkin and Rao for uniform multicommodity flow in planar graphs. This work appears in [3].

Continuing work has recently involved exploring decomposition methods for high-degree constant expansion graphs. Some progress has been made in particular on (considerable) shortening of the proofs of Räcke's celebrated result. Most recently, the authors have found a direct decomposition into well-linked sets that avoids the usef of Räcke's decomposition. One interesting side-effect is that it allows the authors to obtain similar polylog-approximation algorithms for the node-disjoint versions of the problem. This is not possible by Räcke's decomposition since in that case $\Omega(\sqrt{n})$ gaps are known in the node versions of oblivious routing. This work has been accepted to STOC 2005 in a paper [6] *Multicommodity flow, well-linked terminals, and routing problems.*

Work on Hilbert Bases by Andrew McGregor, Bruce Shepherd

While McGregor has been primarily working on coding theory and approximation algorithm, he decided tô spend the summer learning more about the basic theory of math programming. Supplied with the book of Schrijver and Cornuéjols, he has learned most of the basic definitions (face, cones, Hilbert bases, branch and bound) quickly. The two worked on one of the 10 celebrated conjectures in Cornujols book: that in order to check total dual integrality of an

ideal matrix, it is enough to check each $0, 1, \infty$ valued objective. Preempted by subproblems in this effort, the two spent most of their time examining the integrality and toal dual integrality of problems of the form: $P^*(A) = \{x : Ax \leq 1\}$, for $0, 1$ A. While the theory of such problems is well known in the presence of nonnegativity constraints, there does not appear (yet!) to be literature on the version where negativity is allowed. Currently we refer to a matrix as P* if $P^*(A)$ is integral, and TDI-P* if the corresponding system is TDI. We have noted that P* matrices include balanced and $0, 1$ TUM matrices, and are not included in the class of perfect matrices. We are currently exploring a conjecture about when a matrix is TDI-P* based on whether the rows of A form a Hilbert basis.

Work on Robust Optimization by C. Chekuri, G. Oriolo. M.G. Scutella, and Bruce Shepherd

During Gianpaolo Oriolo's visit to Bell Labs, the authors explored the problem of robust network design.

Network designers have traditionally adopted the view that an accurate estimate for point-to-point traffic is given a priori. With the increasing importance of flexible services (such as VPNs or remote storage/computing), there has been increasing interest in the design of networks for situations where traffic patterns are either not well known a priori or changing rapidly. In these settings the network should be dimensioned to support not just one traffic matrix, but a larger class of matrices determined by the application. This results in a *robust optimization* problem, where we are given a *universe* $\mathcal{U}$ of demand matrices (normally specified as a convex region), and the goal is to design the network so that every demand matrix in $\mathcal{U}$ can be *supported* at the lowest possible cost. The simplest form of this problem, was recently shown to be NP-hard [5] by Chekuri, Oriolo, Scutella and Shepherd. The problem they consider is how to allocate fractional link capacities that are sufficient to support every demand, i.e., so that there is a multicommodity flow for every demand matrix in $\mathcal{U}$. It is perhaps the fact that capacities are allowed to be fractional that makes this result somewhat surprising.

In order to give a reduction they introduce a "trick" of considering a more simply analyzed class $\mathcal{U}$ of matrices they call *single-source hose demand matrices*. Here, there is a root node r with a specified marginal value b_r, and each other node also has a marginal value b_v. They normally consider that b_v's are $0, 1$ and that b_r is some integer. A matrix d is then a single-hose demand if $d_{ij} = 0$ unless $i = r$ and $\sum_j d_{rj} \leq b_r$ and each $d_{ri} \leq b_i$. We then ask for the minimum fractional capacity so that every such single-source demand matrix can be routed.

In the case, where all b_v's are 1 and b_r is also, then the optimal fractional capacity allocation is obviously just a minimum spanning tree. If only some of the b_v's are 1, then it is the fractional relaxation for the undirected Steiner tree problem. In the case, where all b_v's are 1, and $b_r = n$, then the optimal solution

is a shortest path tree rooted at r. So at the extreme values of b_r, the problem is easy. The authors show that the problem becomes hard for "middle" values, e.g., $b_r = n/2$. In this case, they give a short reduction from checking whether a graph has the expander property.

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